SCALAR PRODUCT OF VECTORS

OBJECTIVES

1. $(\mathbf{r}.\mathbf{i})^2 + (\mathbf{r}.\mathbf{j})^2 + (\mathbf{r}.\mathbf{k})^2 =$

(a) $3r^2$ (b) r^2

(c) 0 (d) None of these

2. If a, b, c are mutually perpendicular unit vectors, then $|\mathbf{a}+\mathbf{b}+\mathbf{c}| =$

(a) $\sqrt{3}$ (b) 3

(c) 1 (d) 0

3. If vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ **satisfy the condition** $|\mathbf{a}-\mathbf{c}| = |\mathbf{b}-\mathbf{c}|$**, then** $(\mathbf{b}-\mathbf{a}).\left(\mathbf{c}-\frac{\mathbf{a}+\mathbf{b}}{2}\right)$ **is equal to**

(a) 0 (b) –1

(c) 1 (d) 2

4. If $|\mathbf{a}| = 3, |\mathbf{b}| = 1, |\mathbf{c}| = 4$ **and** $\mathbf{a}+\mathbf{b}+\mathbf{c} = \mathbf{0}$, **then** $\mathbf{a}.\mathbf{b}+\mathbf{b}.\mathbf{c}+\mathbf{c}.\mathbf{a} =$

(a) – 13 (b) – 10

(c) 13 (d) 10

5. If $\mathbf{a} = \mathbf{i}+2\mathbf{j}-3\mathbf{k}$ **and** $\mathbf{b} = 3\mathbf{i}-\mathbf{j}+2\mathbf{k}$, **then the angle between the vectors** $\mathbf{a}+\mathbf{b}$ **and** $\mathbf{a}-\mathbf{b}$ **is**

(a) 30^o (b) 60^o

(c) 90^o (d) 0^o

6. If θ **be the angle between the unit vectors a and b, then** $\cos\frac{\theta}{2} =$

(a) $\frac{1}{2}|\mathbf{a}-\mathbf{b}|$ (b) $\frac{1}{2}|\mathbf{a}+\mathbf{b}|$

(c) $\frac{|\mathbf{a}-\mathbf{b}|}{|\mathbf{a}+\mathbf{b}|}$ (d) $\frac{|\mathbf{a}+\mathbf{b}|}{|\mathbf{a}-\mathbf{b}|}$

7. A vector whose modulus is $\sqrt{51}$ **and makes the same angle with** $\mathbf{a} = \frac{\mathbf{i}-2\mathbf{j}+2\mathbf{k}}{3}$, $\mathbf{b} = \frac{-4\mathbf{i}-3\mathbf{k}}{5}$ **and** $\mathbf{c} = \mathbf{j}$, will be

(a) $5\mathbf{i}+5\mathbf{j}+\mathbf{k}$ (b) $5\mathbf{i}+\mathbf{j}-5\mathbf{k}$

(c) $5\mathbf{i}+\mathbf{j}+5\mathbf{k}$ (d) $\pm(5\mathbf{i}-\mathbf{j}-5\mathbf{k})$

8. **Let a, b and c be vectors with magnitudes 3, 4 and 5 respectively and a + b + c = 0, then the values**

(a) 47 (b) 25

(c) 50 (d) – 25

9. **If in a right angled triangle *ABC*, the hypotenuse** $AB = p$, **then** $\overrightarrow{AB} \, . \, \overrightarrow{AC} + \overrightarrow{BC} \, . \, \overrightarrow{BA} + \overrightarrow{CA} \, . \, \overrightarrow{CB}$ **is equal to**

(a) $2p^2$ (b) $\frac{p^2}{2}$

(c) p^2 (d) None of these

10. **The horizontal force and the force inclined at an angle** 60^o **with the vertical, whose resultant is in vertical direction of *P kg*, are**

(a) P, $2P$ (b) $P, \, P\sqrt{3}$

(c) $2P, \, P\sqrt{3}$ (d) None of these

11. **If a is any vector in space, then**

(a) $\mathbf{a} = (\mathbf{a}\,.\,\mathbf{i})\mathbf{i} + (\mathbf{a}\,.\,\mathbf{j})\mathbf{j} + (\mathbf{a}\,.\,\mathbf{k})\mathbf{k}$

(b) $\mathbf{a} = (\mathbf{a} \times \mathbf{i}) + (\mathbf{a} \times \mathbf{j}) + (\mathbf{a} \times \mathbf{k})$

(c) $\mathbf{a} = \mathbf{j}(\mathbf{a}\,.\,\mathbf{i}) + \mathbf{k}(\mathbf{a}\,.\,\mathbf{j}) + \mathbf{i}(\mathbf{a}\,.\,\mathbf{k})$

(d) $\mathbf{a} = (\mathbf{a} \times \mathbf{i}) \times \mathbf{i} + (\mathbf{a} \times \mathbf{j}) \times \mathbf{j} + (\mathbf{a} \times \mathbf{k}) \times \mathbf{k}$

12. **A unit vector which is coplanar to vector** $\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ **and** $\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ **and perpendicular to** $\mathbf{i} + \mathbf{j} + \mathbf{k}$, **is**

(a) $\frac{\mathbf{i} - \mathbf{j}}{\sqrt{2}}$ (b) $\pm\left(\frac{\mathbf{j} - \mathbf{k}}{\sqrt{2}}\right)$

(c) $\frac{\mathbf{k} - \mathbf{i}}{\sqrt{2}}$ (d) $\frac{\mathbf{i} + \mathbf{j} + \mathbf{k}}{\sqrt{3}}$

13. **If *ABCDEF* is regular hexagon, the length of whose side is *a*, then** $\overrightarrow{AB} \, . \, \overrightarrow{AF} + \frac{1}{2}\overrightarrow{BC}^2 =$

(a) a (b) a^2 (c) $2a^2$ (d)0

14. **If the angle between a and b be** 30^o**, then the angle between 3 a and – 4 b will be**

(a) 150^o (b) 90^o

(c) 120^o (d) 30^o

15. If the angle between two vectors $\mathbf{i}+\mathbf{k}$ and $\mathbf{i}-\mathbf{j}+a\mathbf{k}$ is $\pi/3$, then the value of $a=$

(a)2 (b)4

(c) – 2 (d) 0

16. If $|\mathbf{a}|=3, |\mathbf{b}|=4, |\mathbf{c}|=5$ and $\mathbf{a}+\mathbf{b}+\mathbf{c}=0$, then the angle between a and b is

(a) 0 (b) $\frac{\pi}{6}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

17. $\mathbf{a},\mathbf{b},\mathbf{c}$ are three vectors, such that $\mathbf{a}+\mathbf{b}+\mathbf{c}=0$, $|\mathbf{a}|=1, |\mathbf{b}|=2, |\mathbf{c}|=3$, then $\mathbf{a}.\mathbf{b}+\mathbf{b}.\mathbf{c}+\mathbf{c}.\mathbf{a}$ is equal to

(a) 0 (b) – 7

(c) 7 (d) 1

18. If a, b, c are non-zero vectors such that $\mathbf{a}\,.\,\mathbf{b}=\mathbf{a}\,.\,\mathbf{c}$, then which statement is true

(a) **b** = **c** (b) $\mathbf{a}\perp(\mathbf{b}-\mathbf{c})$

(c) $\mathbf{b}=\mathbf{c}$ or $\mathbf{a}\perp(\mathbf{b}-\mathbf{c})$ (d)None of these

19. If $\mathbf{p}=\mathbf{i}-2\mathbf{j}+3\mathbf{k}$ and $\mathbf{q}=3\mathbf{i}+\mathbf{j}+2\mathbf{k}$, then a vector along r which is linear combination of p and q and also perpendicular to q is

(a) $\mathbf{i}+5\mathbf{j}-4\mathbf{k}$ (b) $\mathbf{i}-5\mathbf{j}+4\mathbf{k}$

(c) $-\frac{1}{2}(\mathbf{i}+5\mathbf{j}-4\mathbf{k})$ (d)None of these

20. If a, b, c are three vectors such that $\mathbf{a}=\mathbf{b}+\mathbf{c}$ and the angle between b and c is $\pi/2$, then

(a) $a^2=b^2+c^2$ (b) $b^2=c^2+a^2$

(c) $c^2=a^2+b^2$ **(d)** $2a^2-b^2=c^2$

21. The value of *x* for which the angle between the vectors $\mathbf{a}=-3\mathbf{i}+x\mathbf{j}+\mathbf{k}$ and $\mathbf{b}=x\mathbf{i}+2x\mathbf{j}+\mathbf{k}$ is acute and the angle between b and *x*-axis lies between $\pi/2$ and π satisfy

(a) $x>0$ (b) $x<0$

(c) $x>1$ Only (d) $x<-1$ only

22. *A, B, C, D* are any four points, then

$\overrightarrow{AB}\,.\,\overrightarrow{CD}+\overrightarrow{BC}\,.\,\overrightarrow{AD}+\overrightarrow{CA}\,.\,\overrightarrow{BD}=$

(a) $2\,\overrightarrow{AB}\,.\,\overrightarrow{BC}\,.\,\overrightarrow{CD}$ (b) $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CD}$

(c) $5\sqrt{3}$ (d)0

23. If a, b, c are unit vectors such that $\mathbf{a}+\mathbf{b}+\mathbf{c}=\mathbf{0}$, then $\mathbf{a}\,.\,\mathbf{b}+\mathbf{b}\,.\,\mathbf{c}+\mathbf{c}\,.\,\mathbf{a}=$

(a) 1 (b) 3 (c)– 3/2 (d)3/2

24. **The angle between the vectors $\mathbf{i}-\mathbf{j}+\mathbf{k}$ and $\mathbf{i}+2\mathbf{j}+\mathbf{k}$ is**

(a) $\cos^{-1}\left(\frac{1}{\sqrt{15}}\right)$ (b) $\cos^{-1}\left(\frac{4}{\sqrt{15}}\right)$

(c) $\cos^{-1}\left(\frac{4}{15}\right)$ (d) $\frac{\pi}{2}$

25. **If $\mathbf{d}=\lambda(\mathbf{a}\times\mathbf{b})+\mu(\mathbf{b}\times\mathbf{c})+\nu(\mathbf{c}\times\mathbf{a})$ and $[\mathbf{a}\,\mathbf{b}\,\mathbf{c}]=\frac{1}{8}$, then $\lambda+\mu+\nu$ is equal to**

(a) $8\mathbf{d}.(\mathbf{a}+\mathbf{b}+\mathbf{c})$ (b) $8\mathbf{d}\times(\mathbf{a}+\mathbf{b}+\mathbf{c})$

(c) $\frac{\mathbf{d}}{8}.(\mathbf{a}+\mathbf{b}+\mathbf{c})$ (d) $\frac{\mathbf{d}}{8}\times(\mathbf{a}+\mathbf{b}+\mathbf{c})$

26. **$\mathbf{a},\mathbf{b}$ and c are three vectors with magnitude $|\mathbf{a}|=4$, $|\mathbf{b}|=4$, $|\mathbf{c}|=2$ and such that $\mathbf{a}$ is perpendicular to $(\mathbf{b}+\mathbf{c}),\mathbf{b}$ is perpendicular to $(\mathbf{c}+\mathbf{a})$ and $\mathbf{c}$ is perpendicular to $(\mathbf{a}+\mathbf{b})$. It follows that $|\mathbf{a}+\mathbf{b}+\mathbf{c}|$ is equal to**

(a) 9 (b) 6

(c) 5 (d) 4

27. **If a, b and c are unit vectors such that $\mathbf{a}+\mathbf{b}-\mathbf{c}=0$, then the angle between a and b is**

(a) $\pi/6$ (b) $\pi/3$

(c) $\pi/2$ (d) $2\pi/3$

28. **If $\vec{\lambda}$ is a unit vector perpendicular to plane of vector a and b and angle between them is θ, then a . b will be**

(a) $|\mathbf{a}||\mathbf{b}|\sin\theta\,\vec{\lambda}$ (b) $|\mathbf{a}||\mathbf{b}|\cos\theta\,\vec{\lambda}$

(c) $|\mathbf{a}||\mathbf{b}|\cos\theta$ (d) $|\mathbf{a}||\mathbf{b}|\sin\theta$

29. **If three vectors a, b, c satisfy $\mathbf{a}+\mathbf{b}+\mathbf{c}=\mathbf{0}$ and $|\mathbf{a}|=3$, $|\mathbf{b}|=5$, $|\mathbf{c}|=7$, then the angle between a and b is**

(a) 30^o (b) 45^o

(c) 60^o (d) 90^o

30. **If $\mathbf{a}=4\mathbf{i}+6\mathbf{j}$ and $\mathbf{b}=3\mathbf{j}+4\mathbf{k}$, then the component of a along b is**

(a) $\frac{18}{10\sqrt{3}}(3\mathbf{j}+4\mathbf{k})$ (b) $\frac{18}{25}(3\mathbf{j}+4\mathbf{k})$

(c) $\frac{18}{\sqrt{3}}(3\mathbf{j}+4\mathbf{k})$ (d) $(3\mathbf{j}+4\mathbf{k})$

31. Let a and b be two unit vectors inclined at an angle θ, then $\sin(\theta/2)$ is equal to

(a) $\frac{1}{2}|\mathbf{a}-\mathbf{b}|$ (b) $\frac{1}{2}|\mathbf{a}+\mathbf{b}|$

(c) $|\mathbf{a}-\mathbf{b}|$ (d) $|\mathbf{a}+\mathbf{b}|$

32. The vectors $2\mathbf{i}+3\mathbf{j}-4\mathbf{k}$ and $a\mathbf{i}+b\mathbf{j}+c\mathbf{k}$ are perpendicular, when

(a) $a=2,\ b=3,\ c=-4$ (b) $a=4,\ b=4,\ c=5$

(c) $a=4,\ b=4,\ c=-5$ (d) None of these

33. The projection of vector $2\mathbf{i}+3\mathbf{j}-2\mathbf{k}$ on the vector $\mathbf{i}+2\mathbf{j}+3\mathbf{k}$ will be

(a) $\frac{1}{\sqrt{14}}$ (b) $\frac{2}{\sqrt{14}}$

(c) $\frac{3}{\sqrt{14}}$ (d) $\sqrt{14}$

34. The projection of the vector $\mathbf{i}-2\mathbf{j}+\mathbf{k}$ on the vector $4\mathbf{i}-4\mathbf{j}+7\mathbf{k}$

(a) $\frac{5\sqrt{6}}{10}$ (b) $\frac{19}{9}$

(c) $\frac{9}{19}$ (d) $\frac{\sqrt{6}}{19}$

35. If $\mathbf{a}\neq\mathbf{0},\ \mathbf{b}\neq\mathbf{0}$ and $|\mathbf{a}+\mathbf{b}|=|\mathbf{a}-\mathbf{b}|$, then the vectors a and b are

(a) Parallel to each other

(b) Perpendicular to each other

(c) Inclined at an angle of 60^o

(d) Neither perpendicular nor parallel

36. If the vectors $a\mathbf{i}+b\mathbf{j}+c\mathbf{k}$ and $p\mathbf{i}+q\mathbf{j}+r\mathbf{k}$ are perpendicular, then

(a) $(a+b+c)(p+q+r)=0$ (b) $(a+b+c)(p+q+r)=1$

(c) $ap+bq+cr=0$ (d) $ap+bq+cr=1$

37. The angle between the vector $2\mathbf{i}+3\mathbf{j}+\mathbf{k}$ and $2\mathbf{i}-\mathbf{j}-\mathbf{k}$ is

(a) $\pi/2$ (b) $\pi/4$

(c) $\pi/3$ (d) 0

38. If $l\mathbf{a}+m\mathbf{b}+n\mathbf{c}=\mathbf{0}$, where l, m, n are scalars and a, b, c are mutually perpendicular vectors, then

(a) $l=m=n=1$ (b) $l+m+n=1$

(c) $l=m=n=0$ (d) $l\neq 0,\ m\neq 0,\ n\neq 0$

39. If vector $\mathbf{a} = 2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$ **and vector** $\mathbf{b} = -2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, **then** $\frac{\text{Projection of vector } \mathbf{a} \text{ on vector } \mathbf{b}}{\text{Projection of vector } \mathbf{b} \text{ on vector } \mathbf{a}} =$

(a) $\frac{3}{7}$ (b) $\frac{7}{3}$

(c) 3 (d) 7

40. If a and b are two unit vectors such that $\mathbf{a} + 2\mathbf{b}$ **and** $5\mathbf{a} - 4\mathbf{b}$ **are perpendicular to each other, then the angle between a and b**

(a) 45^o (b) 60^o

(c) $\cos^{-1}\left(\frac{1}{3}\right)$ (d) $\cos^{-1}\left(\frac{2}{7}\right)$

41. The unit normal vector to the line joining $\mathbf{i} - \mathbf{j}$ **and** $2\mathbf{i} + 3\mathbf{j}$ **pointing towards the origin is**

(a) $\frac{4\mathbf{i} - \mathbf{j}}{\sqrt{17}}$ (b) $\frac{-4\mathbf{i} + \mathbf{j}}{\sqrt{17}}$

(c) $\frac{2\mathbf{i} - 3\mathbf{j}}{\sqrt{13}}$ (d) $\frac{-2\mathbf{i} + 3\mathbf{j}}{\sqrt{13}}$

42. If $\mathbf{a} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ **and** $\mathbf{b} = 5\mathbf{i} - 3\mathbf{j} + \mathbf{k}$, **then the projection of b on a is**

(a) 3 (b) 4

(c) 5 (d) 6

43. If in a right angled triangle ***ABC*****, the hypotenuse** $AB = p$, **then** $\overrightarrow{AB} \cdot \overrightarrow{AC} + \overrightarrow{BC} \cdot \overrightarrow{BA} + \overrightarrow{CA} \cdot \overrightarrow{CB}$ **is equal to**

(a) $2p^2$ (b) $\frac{p^2}{2}$ (c) p^2 (d)None of these

44. If a, b, c are non-zero vectors such that $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$, **then which statement is true**

(a)**b** = **c** (b) $\mathbf{a} \perp (\mathbf{b} - \mathbf{c})$

(c) $\mathbf{b} = \mathbf{c}$ Or $\mathbf{a} \perp (\mathbf{b} - \mathbf{c})$ (d) None of these

45. If $|\mathbf{a} + \mathbf{b}| > |\mathbf{a} - \mathbf{b}|$, **then the angle between a and b is**

(a) Acute (b) Obtuse

(c) $\frac{\pi}{2}$ (d) π

SCALAR PRODUCT OF VECTORS

HINTS AND SOLUTIONS

1. (b) Let $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} \Rightarrow \mathbf{r}.\mathbf{i} = x, \ \mathbf{r}.\mathbf{j} = y, \ \mathbf{r}.\mathbf{k} = z$

$\Rightarrow (\mathbf{r}.\mathbf{i})^2 + (\mathbf{r}.\mathbf{j})^2 + (\mathbf{r}.\mathbf{k})^2 = x^2 + y^2 + z^2 = r^2$.

2. (a) Three mutually perpendicular unit vectors $= \mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$.

Therefore $|\mathbf{a}| = |\mathbf{b}| = |\mathbf{c}| = 1$ and $\mathbf{a}.\mathbf{b} = \mathbf{b}.\mathbf{c} = \mathbf{c}.\mathbf{a} = 0$.

We know that

$|\mathbf{a}+\mathbf{b}+\mathbf{c}|^2 = (\mathbf{a}+\mathbf{b}+\mathbf{c}).(\mathbf{a}+\mathbf{b}+\mathbf{c}) = |\mathbf{a}|^2 + |\mathbf{b}|^2 + |\mathbf{c}|^2 + 2(\mathbf{a}.\mathbf{b} + \mathbf{b}.\mathbf{c} + \mathbf{c}.\mathbf{a}) = 1+1+1+0 = 3$

Or $|\mathbf{a}+\mathbf{b}+\mathbf{c}| = \sqrt{3}$.

3. (a) $(\mathbf{b}-\mathbf{a}).\left(\mathbf{c} - \frac{\mathbf{a}+\mathbf{b}}{2}\right) = \mathbf{b}.\mathbf{c} - \mathbf{b}.\left(\frac{\mathbf{a}+\mathbf{b}}{2}\right) - \mathbf{a}.\mathbf{c} + \frac{\mathbf{a}}{2}(\mathbf{a}+\mathbf{b})$

and $|\mathbf{a}-\mathbf{c}| = |\mathbf{b}-\mathbf{c}| \Rightarrow |\mathbf{a}-\mathbf{c}|^2 = |\mathbf{b}-\mathbf{c}|^2$ $\therefore$ $\mathbf{a}+\mathbf{b} = 2\mathbf{c}$

Therefore, $(\mathbf{b}-\mathbf{a}).\left(\mathbf{c} - \frac{\mathbf{a}+\mathbf{b}}{2}\right) = 0$.

4. (a) $(\mathbf{a}+\mathbf{b}+\mathbf{c})^2 = 0$

$\Rightarrow |\mathbf{a}|^2 + |\mathbf{b}|^2 + |\mathbf{c}|^2 + 2\mathbf{a}.\mathbf{b} + 2\mathbf{b}.\mathbf{c} + 2\mathbf{c}.\mathbf{a} = 0$

$\Rightarrow 9 + 1 + 16 + 2(\mathbf{a}.\mathbf{b} + \mathbf{b}.\mathbf{c} + \mathbf{c}.\mathbf{a}) = 0$

$\Rightarrow \mathbf{a}.\mathbf{b} + \mathbf{b}.\mathbf{c} + \mathbf{c}.\mathbf{a} = -\frac{26}{2} = -13$.

5. (c) $\mathbf{a}+\mathbf{b} = 4\mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{a}-\mathbf{b} = -2\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}$.

$(\mathbf{a}+\mathbf{b}).(\mathbf{a}-\mathbf{b}) = 0$. Hence $(\mathbf{a}+\mathbf{b}) \perp (\mathbf{a}-\mathbf{b})$.

6. (b) $(\mathbf{a}+\mathbf{b}).(\mathbf{a}+\mathbf{b}) = |\mathbf{a}|^2 + |\mathbf{b}|^2 + 2\mathbf{a}.\mathbf{b}$ Or $|\mathbf{a}+\mathbf{b}|^2 = 2.2\cos^2\frac{\theta}{2} \Rightarrow \cos\frac{\theta}{2} = \frac{1}{2}|\mathbf{a}+\mathbf{b}|$.

7. (d) Verification

8. (d) $\because$ $\mathbf{a}+\mathbf{b}+\mathbf{c} = 0 \Rightarrow (\mathbf{a}+\mathbf{b}+\mathbf{c})^2 = 0$

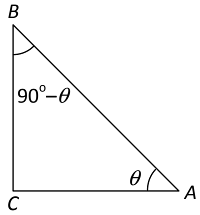
$|\mathbf{a}|^2 + |\mathbf{b}|^2 + |\mathbf{c}|^2 + 2(\mathbf{a}.\mathbf{b} + \mathbf{b}.\mathbf{c} + \mathbf{c}.\mathbf{a}) = 0$

$\Rightarrow 2(\mathbf{a}.\mathbf{b} + \mathbf{b}.\mathbf{c} + \mathbf{c}.\mathbf{a}) = -(9 + 16 + 25)$

$\Rightarrow \mathbf{a}.\mathbf{b} + \mathbf{b}.\mathbf{c} + \mathbf{c}.\mathbf{a} = -25$.

9. (c) We have $\overrightarrow{AB}.\overrightarrow{AC}+\overrightarrow{BC}.\overrightarrow{BA}+\overrightarrow{CA}.\overrightarrow{CB}$

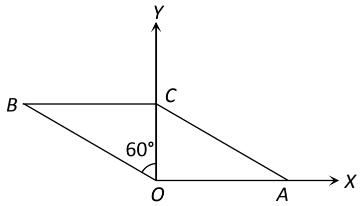
$(AB)(AC)\cos\theta+(BC)(BA)\cos(90^o-\theta)+0$



$= AB(AC\cos\theta+BC\sin\theta)=AB\left(\frac{(AC)^2}{AB}+\frac{(BC)^2}{AB}\right)$

$= AC^2+BC^2=AB^2=p^2$.

10. (c) Let $\overrightarrow{OA}=P_1\mathbf{i},\ \overrightarrow{CB}=-P_1\mathbf{i},\ \overrightarrow{OB}=-P_1\mathbf{i}+P\mathbf{j}$



$\frac{\overrightarrow{OB}.\mathbf{j}}{OB}=\cos 60^\circ \Rightarrow \frac{(-P_1\mathbf{i}+P\mathbf{j}).\mathbf{j}}{\sqrt{P_1^2+P^2}}=\frac{1}{2}$

$\Rightarrow 2P=\sqrt{P^2+P_1^2}\Rightarrow P_1=P\sqrt{3}$

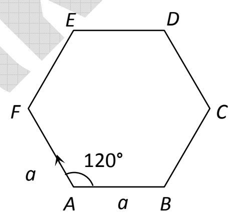
$|\overrightarrow{OB}|=\sqrt{P^2+P_1^2}=\sqrt{P^2+3P^2}=2P.$

11. (a) Let $\mathbf{a}=a_1\mathbf{i}+a_2\mathbf{j}+a_3\mathbf{k}$, then $\mathbf{a}.\mathbf{i}=a_1,\ \mathbf{a}.\mathbf{j}=a_2,\ \mathbf{a}.\mathbf{k}=a_3$

$\therefore\ \mathbf{a}=(\mathbf{a}.\mathbf{i})\mathbf{i}+(\mathbf{a}.\mathbf{j})\mathbf{j}+(\mathbf{a}.\mathbf{k})\mathbf{k}$.

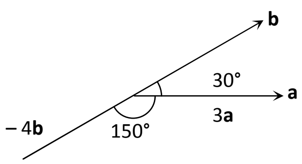
12. (b) Verification

13. (d) $\overrightarrow{AB}.\overrightarrow{AF}=|\mathbf{a}||\mathbf{a}|\cos 120^\circ=\frac{-1}{2}a^2$ and $\frac{1}{2}\overrightarrow{BC}^2=\frac{1}{2}a^2$



Therefore, $\overrightarrow{AB}.\overrightarrow{AF}+\frac{1}{2}\overrightarrow{BC}^2=\frac{1}{2}a^2-\frac{1}{2}a^2=0.$

14. (a) It is obvious from figure.



15. (d) $\cos\frac{\pi}{3} = \frac{1+a}{\sqrt{2}\sqrt{2+a^2}} \Rightarrow a = 0$.

16. (d) $\mathbf{a}+\mathbf{b} = -\mathbf{c} \Rightarrow |\mathbf{a}|^2 + |\mathbf{b}|^2 + 2|\mathbf{a}||\mathbf{b}|\cos\theta = |\mathbf{c}|^2$

$\Rightarrow \cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2}$.

17. (b) $\mathbf{a}+\mathbf{b}+\mathbf{c} = 0 \Rightarrow (\mathbf{a}+\mathbf{b}+\mathbf{c}).(\mathbf{a}+\mathbf{b}+\mathbf{c}) = \mathbf{0}$

$\Rightarrow |\mathbf{a}|^2 + |\mathbf{b}|^2 + |\mathbf{c}|^2 + 2(\mathbf{a.b} + \mathbf{b.c} + \mathbf{c.a}) = 0$

$\Rightarrow \mathbf{a.b} + \mathbf{b.c} + \mathbf{c.a} = \frac{-1-4-9}{2} = -7.$

18. (c) $\mathbf{a.b} = \mathbf{a.c} \Rightarrow \mathbf{a.b} - \mathbf{a.c} = 0 \Rightarrow \mathbf{a}.(\mathbf{b}-\mathbf{c}) = 0$

$\Rightarrow$ Either $\mathbf{b}-\mathbf{c} = \mathbf{0}$ or $\mathbf{a} = \mathbf{0} \Rightarrow \mathbf{b} = \mathbf{c}$ or $\mathbf{a}\perp(\mathbf{b}-\mathbf{c})$.

19. (c) $\mathbf{r} = \mathbf{p} + \lambda\mathbf{q} \Rightarrow \mathbf{r}.\mathbf{q} = \mathbf{p}.\mathbf{q} + \lambda\mathbf{q}.\mathbf{q}$

$\Rightarrow 0 = 7 + 14\lambda \Rightarrow \lambda = -\frac{1}{2}$

Therefore, $\mathbf{r} = -\frac{1}{2}(\mathbf{i} + 5\mathbf{j} - 4\mathbf{k})$.

20. (a) Given that $\mathbf{a} = \mathbf{b} + \mathbf{c}$ and angle between **b** and **c** is $\frac{\pi}{2}$.

So, $\mathbf{a}^2 = \mathbf{b}^2 + \mathbf{c}^2 + 2\mathbf{b}.\mathbf{c}$

Or $\mathbf{a}^2 = \mathbf{b}^2 + \mathbf{c}^2 + 2|\mathbf{b}||\mathbf{c}|\cos\frac{\pi}{2}$

Or $\mathbf{a}^2 = \mathbf{b}^2 + \mathbf{c}^2 + 0$, $\therefore \mathbf{a}^2 = \mathbf{b}^2 + \mathbf{c}^2$

i.e., $a^2 = b^2 + c^2$.

21. (b) For acute angle $\mathbf{a}.\mathbf{b} > 0$

i.e., $-3x + 2x^2 + 1 > 0 \Rightarrow (x-1)(2x-1) > 0$

For obtuse angle between $\mathbf{b}$ and x-axis $\mathbf{b}.\mathbf{i} < 0$

$\Rightarrow x < 0$.

22. (d) $\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} = \mathbf{a} + \mathbf{b} + \mathbf{c}$

$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \mathbf{a} + \mathbf{b}$ Or $\overrightarrow{CA} = -(\mathbf{a} + \mathbf{b})$

$\overrightarrow{BD} = \overrightarrow{BC} + \overrightarrow{CD} = \mathbf{b} + \mathbf{c}$

Therefore, $\overrightarrow{AB}.\overrightarrow{CD} + \overrightarrow{BC}.\overrightarrow{AD} + \overrightarrow{CA}.\overrightarrow{BD}$

$= \mathbf{a}.\mathbf{c} + \mathbf{b}.(\mathbf{a}+\mathbf{b}+\mathbf{c}) + (-\mathbf{a}-\mathbf{b}).(\mathbf{b}+\mathbf{c})$

$= \mathbf{a}.\mathbf{c} + \mathbf{b}.\mathbf{a} + \mathbf{b}.\mathbf{b} + \mathbf{b}.\mathbf{c} - \mathbf{a}.\mathbf{b} - \mathbf{a}.\mathbf{c} - \mathbf{b}.\mathbf{b} - \mathbf{b}.\mathbf{c} = 0$.

23. (c) Squaring $(\mathbf{a}+\mathbf{b}+\mathbf{c})=\mathbf{0}$,

We get $\mathbf{a}^2+\mathbf{b}^2+\mathbf{c}^2+2\mathbf{a}.\mathbf{b}+2\mathbf{b}.\mathbf{c}+2\mathbf{c}.\mathbf{a}=0$

$\Rightarrow |\mathbf{a}|^2+|\mathbf{b}|^2+|\mathbf{c}|^2+2(\mathbf{a}.\mathbf{b}+\mathbf{b}.\mathbf{c}+\mathbf{c}.\mathbf{a})=0$

$\Rightarrow 2(\mathbf{a}.\mathbf{b}+\mathbf{b}.\mathbf{c}+\mathbf{c}.\mathbf{a})=-3 \Rightarrow \mathbf{a}.\mathbf{b}+\mathbf{b}.\mathbf{c}+\mathbf{c}.\mathbf{a}=-\frac{3}{2}$.

24. (d) $(\mathbf{i}-\mathbf{j}+\mathbf{k}).(\mathbf{i}+2\mathbf{j}+\mathbf{k})=\sqrt{3}\sqrt{6}\cos\theta$

$\Rightarrow \cos\theta=\frac{0}{\sqrt{3}\sqrt{6}} \Rightarrow \theta=\frac{\pi}{2}$.

25. (a) $\mathbf{d}.\mathbf{c}=\lambda(\mathbf{a}\times\mathbf{b}).\mathbf{c}+\mu(\mathbf{b}\times\mathbf{c}).\mathbf{c}+\nu(\mathbf{c}\times\mathbf{a}).\mathbf{c}$

$=\lambda[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]+0+0=\lambda[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]=\frac{\lambda}{8}$

Hence $\lambda=8(\mathbf{d}.\mathbf{c})$, $\mu=8(\mathbf{d}.\mathbf{a})$ and $\nu=8(\mathbf{d}.\mathbf{b})$

Therefore, $\lambda+\mu+\nu=8\mathbf{d}.\mathbf{c}+8\mathbf{d}.\mathbf{a}+8\mathbf{d}.\mathbf{b}$

$=8\mathbf{d}.(\mathbf{a}+\mathbf{b}+\mathbf{c})$.

26. (b) Here $|\mathbf{a}|=4$; $|\mathbf{b}|=4$; $|\mathbf{c}|=2$

And $\mathbf{a}.(\mathbf{b}+\mathbf{c})=0 \Rightarrow \mathbf{a}.\mathbf{b}+\mathbf{a}.\mathbf{c}=0$ .....(i)

$\mathbf{b}.(\mathbf{c}+\mathbf{a})=0 \Rightarrow \mathbf{b}.\mathbf{c}+\mathbf{b}.\mathbf{a}=0$ ..... (ii)

$\mathbf{c}.(\mathbf{a}+\mathbf{b})=0 \Rightarrow \mathbf{c}.\mathbf{a}+\mathbf{c}.\mathbf{b}=0$ ..... (iii)

Adding (i), (ii) and (iii), we get, $2[\mathbf{a}.\mathbf{b}+\mathbf{b}.\mathbf{c}+\mathbf{c}.\mathbf{a}]=0$

$\therefore |\mathbf{a}+\mathbf{b}+\mathbf{c}|=\sqrt{|\mathbf{a}|^2+|\mathbf{b}|^2+|\mathbf{c}|^2+2(\mathbf{a}.\mathbf{b}+\mathbf{b}.\mathbf{c}+\mathbf{c}.\mathbf{a})}$

$=\sqrt{|\mathbf{a}|^2+|\mathbf{b}|^2+|\mathbf{c}|^2}=\sqrt{16+16+4}$

$\Rightarrow |\mathbf{a}+\mathbf{b}+\mathbf{c}|=6$.

27. (d) Given condition is $\mathbf{a}+\mathbf{b}=\mathbf{c}$.

Using dot product, $(\mathbf{a}+\mathbf{b}).(\mathbf{a}+\mathbf{b})=\mathbf{c}.\mathbf{c}$

$\Rightarrow \mathbf{a}.\mathbf{a}+\mathbf{b}.\mathbf{b}+2\mathbf{a}.\mathbf{b}=\mathbf{c}.\mathbf{c}$

$\Rightarrow |\mathbf{a}|.|\mathbf{a}|\cos 0°+|\mathbf{b}|.|\mathbf{b}|\cos 0°+2|\mathbf{a}|.|\mathbf{b}|\cos\alpha$

$=|\mathbf{c}|.|\mathbf{c}|\cos 0°$, $(\because |\mathbf{a}|=|\mathbf{b}|=|\mathbf{c}|=1)$

$\Rightarrow 1+1+2\cos\alpha=1 \Rightarrow \cos\alpha=-\frac{1}{2} \Rightarrow \alpha=\frac{2\pi}{3}$.

28. (c) Concept

29. (c) $\mathbf{a}+\mathbf{b}+\mathbf{c}=\mathbf{0} \Rightarrow \mathbf{a}+\mathbf{b}=-\mathbf{c}$

$\Rightarrow |\mathbf{a}|^2 + |\mathbf{b}|^2 + 2|\mathbf{a}||\mathbf{b}|\cos\theta = |-\mathbf{c}|^2$

$\Rightarrow 9 + 25 + 30\cos\theta = 49 \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60°$.

30. (b) The component of vector $\mathbf{a}$ along $\mathbf{b}$ is $\frac{(\mathbf{a}.\mathbf{b})\mathbf{b}}{|\mathbf{b}|^2} = \frac{18}{25}(3\mathbf{j} + 4\mathbf{k})$.

31. (a) $|\mathbf{a} - \mathbf{b}| = \sqrt{1^2 + 1^2 - 2.1^2\cos\theta} = \sqrt{2(1 - \cos\theta)}$

$= \sqrt{2} \times \sqrt{2}\sin\frac{\theta}{2} = 2\sin\frac{\theta}{2} \Rightarrow \sin\frac{\theta}{2} = \frac{|\mathbf{a} - \mathbf{b}|}{2}$.

32. (b) To be perpendicular, $2a + 3b - 4c = 0$ and option (b) satisfies this equation.

33. (b) $(2\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}).\frac{(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})}{\sqrt{14}} = \frac{2}{\sqrt{14}}$.

34. (b) Projection of **a** on **b** $= |\mathbf{a}|\cos\theta = |\mathbf{a}|\frac{\mathbf{a}.\mathbf{b}}{|\mathbf{a}||\mathbf{b}|} = \frac{\mathbf{a}.\mathbf{b}}{|\mathbf{b}|}$

$= \frac{4 + 8 + 7}{\sqrt{16 + 16 + 49}} = \frac{19}{\sqrt{81}} = \frac{19}{9}$.

35. (b) $|\mathbf{a} + \mathbf{b}| = |\mathbf{a} - \mathbf{b}|$; Squaring both sides, we get $4\mathbf{a}.\mathbf{b} = 0 \Rightarrow \mathbf{a}$ is perpendicular to $\mathbf{b}$.

36. (c) It is obvious.

37. (a) Let $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$

Since $\cos\theta = \frac{\mathbf{a}.\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$

$= \frac{(2\mathbf{i} + 3\mathbf{j} + \mathbf{k}).(2\mathbf{i} - \mathbf{j} - \mathbf{k})}{\sqrt{(2)^2 + (3)^2 + (1)^2}\sqrt{(2)^2 + (-1)^2 + (-1)^2}}$

$= \frac{4 - 3 - 1}{\sqrt{(4 + 9 + 1)}\sqrt{(4 + 1 + 1)}} = 0 \qquad \therefore \theta = \frac{\pi}{2}$.

38. (c) $l\mathbf{a} + m\mathbf{b} + n\mathbf{c} = \mathbf{0}$

$\Rightarrow a^2l^2 + m^2b^2 + n^2c^2 + 2lm\,\mathbf{a}.\mathbf{b} + 2ln\,\mathbf{a}.\mathbf{c} + 2mn\,\mathbf{b}.\mathbf{c} = 0$

But $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are mutually perpendicular

So, $\mathbf{a}.\mathbf{b}$, $\mathbf{b}.\mathbf{c}$ and $\mathbf{c}.\mathbf{a}$ are equal to zero.

Therefore, $a^2l^2 + m^2b^2 + n^2c^2 = 0$ *i.e.,* l, m, n are equal to zero because a^2, b^2 and c^2 cannot be equal to zero.

39. (b) Required value $= \frac{\mathbf{b}.\mathbf{a}}{|\mathbf{b}|} \Big/ \frac{\mathbf{a}.\mathbf{b}}{|\mathbf{a}|} = \frac{|\mathbf{a}|}{|\mathbf{b}|} = \frac{7}{3}$.

40. (b) $(\mathbf{a} + 2\mathbf{b}).(5\mathbf{a} - 4\mathbf{b}) = 0$ or $5\mathbf{a}^2 + 6\mathbf{a}.\mathbf{b} - 8\mathbf{b}^2 = 0$

Or $6\,\mathbf{a}.\mathbf{b} = 3, \quad (\because \mathbf{a}^2 = 1, \mathbf{b}^2 = 1)$

$\therefore \mathbf{a}.\mathbf{b} = \frac{1}{2}$ Or $|\mathbf{a}||\mathbf{b}|\cos\theta = \frac{1}{2}$

$\therefore \cos\theta = \frac{1}{2}, \quad \therefore \theta = 60^o.$

41. (b) $\vec{L} = \mathbf{i} + 4\mathbf{j}$

Therefore, vector perpendicular to $\vec{L} = \lambda(4\mathbf{i} - \mathbf{j})$

$\therefore$ Unit vector is $\frac{4\mathbf{i} - \mathbf{j}}{\sqrt{17}}$.

But it points towards origin

$\therefore$ Required vector $= \frac{-4\mathbf{i} + \mathbf{j}}{\sqrt{17}}$.

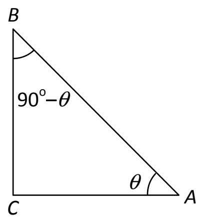
42. (a) Vectors $\mathbf{a} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $\mathbf{b} = 5\mathbf{i} - 3\mathbf{j} + \mathbf{k}$.

We know that the projection of **b** on

$$\mathbf{a} = \frac{\mathbf{b}.\mathbf{a}}{|\mathbf{a}|} = \frac{(2\mathbf{i} + \mathbf{j} + 2\mathbf{k}).(5\mathbf{i} - 3\mathbf{j} + \mathbf{k})}{\sqrt{(2)^2 + (1)^2 + (2)^2}} = \frac{10 - 3 + 2}{\sqrt{9}} = \frac{9}{3} = 3.$$

43. (c)We have $\overrightarrow{AB}.\overrightarrow{AC} + \overrightarrow{BC}.\overrightarrow{BA} + \overrightarrow{CA}.\overrightarrow{CB}$

$(AB)(AC)\cos\theta + (BC)(BA)\cos(90^o - \theta) + 0$



$$= AB(AC\cos\theta + BC\sin\theta) = AB\left(\frac{(AC)^2}{AB} + \frac{(BC)^2}{AB}\right)$$

$= AC^2 + BC^2 = AB^2 = p^2$.

44. (c) $\mathbf{a}.\mathbf{b} = \mathbf{a}.\mathbf{c} \Rightarrow \mathbf{a}.\mathbf{b} - \mathbf{a}.\mathbf{c} = 0 \Rightarrow \mathbf{a}.(\mathbf{b} - \mathbf{c}) = 0$

$\Rightarrow$ Either $\mathbf{b} - \mathbf{c} = \mathbf{0}$ or $\mathbf{a} = \mathbf{0} \Rightarrow \mathbf{b} = \mathbf{c}$ or $\mathbf{a} \perp (\mathbf{b} - \mathbf{c})$.

45. (a) $|\mathbf{a} + \mathbf{b}| > |\mathbf{a} - \mathbf{b}|$

Squaring both sides, we get

$a^2 + b^2 + 2\mathbf{a}.\mathbf{b} > a^2 + b^2 - 2\mathbf{a}.\mathbf{b}$

$\Rightarrow 4\mathbf{a}.\mathbf{b} > 0 \Rightarrow \cos\theta > 0$. Hence $\theta < 90°$, (acute).